Managerial Economics

Chapter 5

Production

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  - Labor productivity during recessions
  - How much will the output produced per worker rise or fall with each additional layoff?
- **Solution Approach**
  - First, a firm must decide how to produce. Second, if a firm wants to expand its output, it must decide how to do that in both the short run and the long run. Third, given its ability to change its output level, a firm must determine how large to grow.
- **Empirical Methods**
  - A production function summarizes how a firm converts inputs into outputs using one available technology, and helps to decide how to produce.
  - Increasing output in the short-run can be done only by increasing variable inputs, but in the long-run there is more flexibility.
  - The size of a firm depends on returns to scale and its growth will be determined by increments in productivity that comes from technological change.

5.1 Production Functions

- Production Process
  - A firm uses a technology or production process to transform inputs or factors of production into outputs.
- Inputs

- Capital (K) - land, buildings, equipment
- Labor (L) – skilled and less-skilled workers
- Materials (M) – natural resources, raw materials, and processed products

- Output
  - It could be a service, such as an automobile tune-up by a mechanic, or a physical product, such as a computer chip or a potato chip

- Production Function
  - Maximum quantity of output that can be produced with different combinations of inputs, given current knowledge about technology and organization
  - A production function shows only efficient production processes because it gives the maximum output.

- $q = f(L, K)$
  - Production function for a firm that uses only labor and capital
  - q units of output (such as wrapped candy bars) are produced using L units of labor services (such as hours of work by assembly-line workers) and K units of capital (such as the number of conveyor belts)

- Time and Variability of Inputs
  - Short run: a period of time so brief that at least one factor of production cannot be varied. Inputs in the short run are fixed or variable inputs.
  - Long run: period of time that all relevant inputs can be varied. Inputs in the long run are all variable.
  - **5.2 Short-Run Production**

The Total Production Function: $q = f(L, \bar{K})$

- Relationship between output and labor when a firm's capital is fixed
- In Table 5.1, capital is fixed to eight fully equipped workbenches. As the number of workers increases, so does output: 1 worker assembles 5 computers in a day, 2 workers assemble 18, 3 workers assemble 36, and so forth.

- The Average Product of Labor: $AP_L = q/L$
  - The ratio of output to the amount of labor used to produce that output

- Table 5.1 shows that 9 workers can assemble 108 computers a day, so the average product of labor for 9 workers is 12 (= 108/9) computers a day.
- Ten workers can assemble 110 computers in a day, so the average product of labor for 10 workers is 11 (= 110/10) computers.
- Thus, increasing the labor force from 9 to 10 workers lowers the average product per worker
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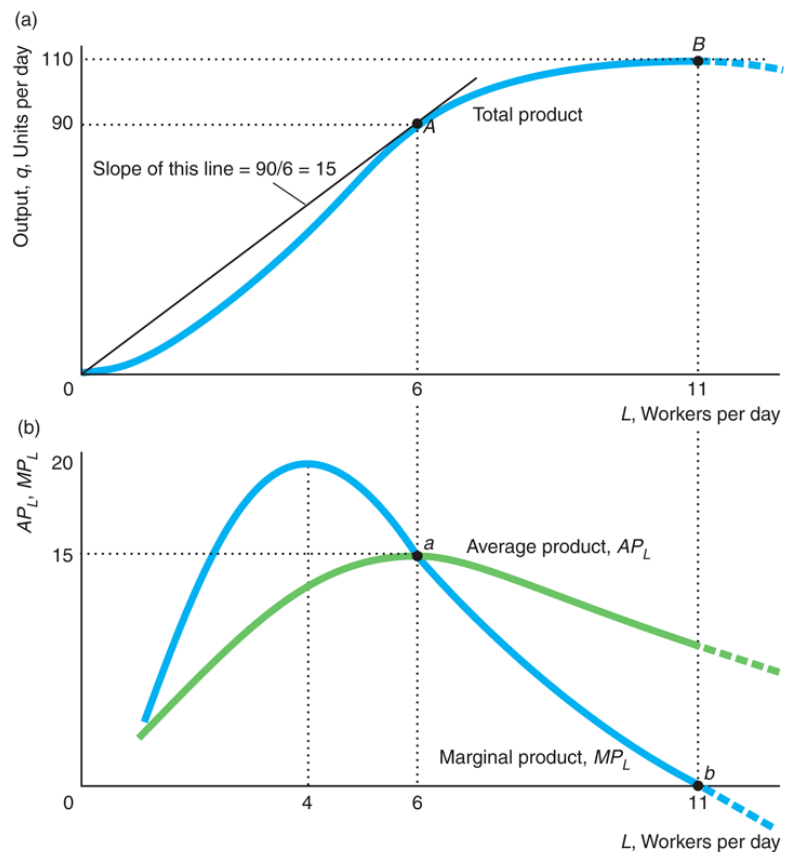
  - Thus, increasing the labor force from 9 to 10 workers lowers the average product per worker
- The Marginal Product of Labor: $MP_L = \Delta q/\Delta L$
  - Change in total output resulting from using an extra unit of labor, holding other factors (capital) constant
  - Table 5.1 shows if the number of workers increases from 1 to 2, $\Delta L = 1$, output rises by $\Delta q = 13 = 18 - 5$, so the marginal product of labor is 13.
  - When the change in labor is very small (infinitesimal) we use the calculus definition of the marginal product of labor: the partial derivative of the production function with respect to labor [$MP_L = \partial q/\partial L = \partial f(L,K)/\partial L$]
- Graphing the Product Curves
  - Figure 5.1 shows how output (total product), the average product of labor, and the marginal product of labor vary with the number of workers.
- Product Curve Characteristics
  - In panel a, output rises with labor until it reaches its maximum of 110 computers at 11 workers, point *C*.

- In panel b, the average product of labor first rises and then falls as labor increases. Also, the marginal product of labor first rises and then falls as labor increases.
- Average product may rise because of division of labor and specialization. Workers become more productive as we add more workers. Marginal product of labor goes up, and consequently average product goes up.
- Average product falls as the number of workers exceeds 6. Workers might have to wait to use equipment or get in each other's way because capital is constant. Because marginal product of labor goes down, average product goes down too.

• **Figure 5.1 Production Relationships with Variable Labor**



• Relationships among Product Curves

- The three curves are geometrically related.

• Average Product of Labor and Marginal Product of Labor

- If the marginal product curve is above that average product curve, the average product must rise with extra labor
- If marginal product is below the average product then the average product must fall with extra labor

- Consequently, the average product curve reaches its peak, where the marginal product and average product are equal (where the curves cross)

- Deriving AP_L and MP_L using the Total Production Function
  - The average product of labor for L workers equals the slope of a straight line from the origin to a point on the total product of labor curve for L workers in panel a.
  - The slope of the total product curve at a given point equals the marginal product of labor. That is, the marginal product of labor equals the slope of a straight line that is tangent to the total output curve at a given point.

- The Law of Diminishing Marginal Returns
  - If a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will eventually become smaller (diminish).
  - This law comes from realizing most observed production functions have this property.
  - This law determines the shape of the marginal product of labor curves: if only one input is increased, the marginal product of that input will diminish eventually.

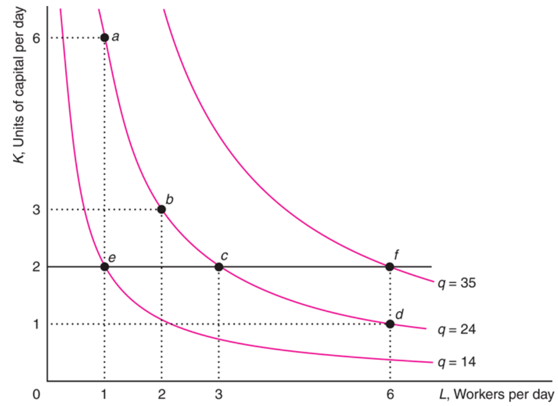
5.3 Long-Run Production

Isoquants: $\bar{q} = f(L,K)$

- In the long run, labor and capital are variable. The firm can substitute one input for another while continuing to produce the same level of output.
- An isoquant shows the efficient combinations of labor and capital that can produce the same (*iso*) level of output (*quantity*). Points *a*, *b*, *c* and *d* show the same level of output, $q = 24$, in Figure 5.2

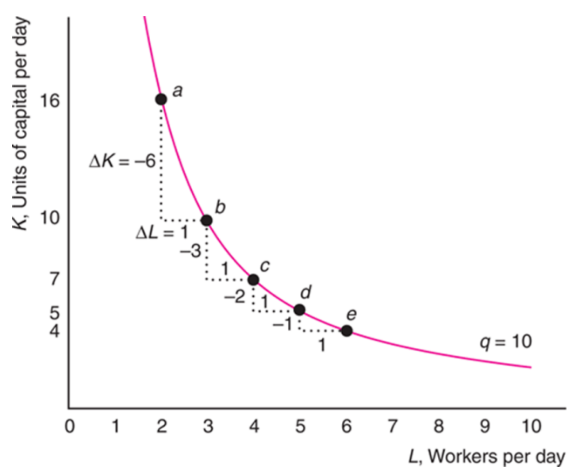
- Properties of Isoquants
  - The farther an isoquant is from the origin, the greater the level of output
  - Isoquants do not cross
  - Isoquants slope downward

- **Figure 5.2 A Family of Isoquants**



- Substituting Inputs: *MRTS*=Δ*K*/Δ*L*
  - The slope of an isoquant shows the ability of a firm to replace one input with another while holding output constant.
  - This slope is the <u>marginal rate of technical substitution</u> (*MRTS*): how many units of capital the firm can replace with an extra unit of labor while holding output constant.
- Diminishing *MRTS* (absolute value)
  - The more labor and less capital the firm has, the harder it is to replace remaining capital with labor and the flatter the isoquant becomes.
  - In Figure 5.4, the firm replaces 6 units of capital per 1 worker to remain on the same isoquant (*a* to *b*), so *MRTS*= -6. If it hires another worker (*b* to *c*), the firm replaces 3 units of capital, *MRTS* = -3.

- **Figure 5.4 How the Marginal Rate of Technical Substitution Varies Along an Isoquant**

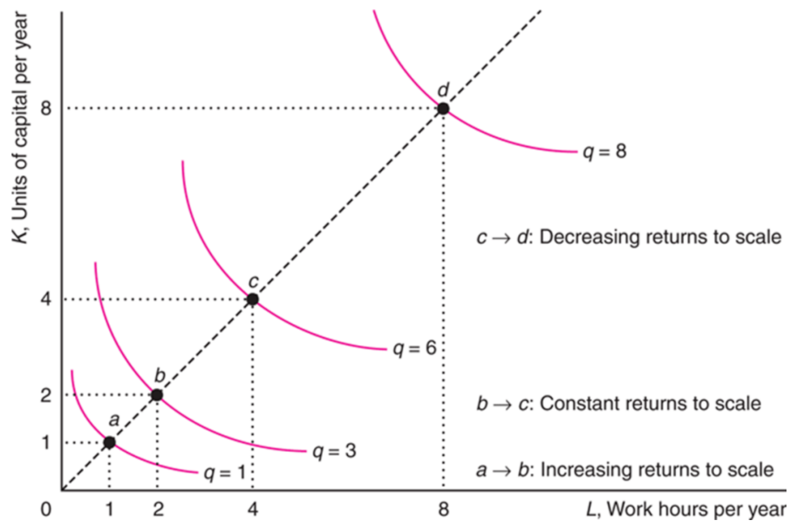


- Substitutability of Inputs and Marginal Products
  - The marginal rate of technical substitution is equal to the ratio of marginal products
  - -*MPL*/*MPK* = Δ*K*/Δ*L* = *MRTS*

- Cobb-Douglas Production Functions: $q = AL^{\alpha}K^{\beta}$
  - A, α, and β are all positive constants
  - The marginal product of labor is $MP_L = \alpha q/L = \alpha AP_L$ *and* $\alpha = MP_L/AP_L$
  - The marginal product of capital is $MP_K = \beta q/K = \beta AP_K$, and $\beta = MP_K/AP_K$
  - $MRTS = -\alpha K/\beta L$

5.4 Returns to Scale

- Constant Returns to Scale (*CRS*): $f(2L, 2K) = 2f(L,K) = 2q$
  - A technology exhibits constant returns to scale if doubling inputs exactly doubles the output. The firm builds an identical second plant and uses the same amount of labor and equipment as in the first plant.
- Increasing Returns to Scale (*IRS*): $f(2L, 2K) > 2f(L,K) = 2q$
  - A technology exhibits increasing returns to scale if doubling inputs more than doubles the output. Instead of building two small plants, the firm decides to build a single larger plant with greater specialization of labor and capital.
- Decreasing Returns to Scale (*DRS*): $f(2L, 2K) < 2f(L,K) = 2q$
  - A technology exhibits decreasing returns to scale if doubling inputs less than doubles output. An owner may be able to manage one plant well but may have trouble organizing, coordinating, and integrating activities in two plants.
- Varying Returns to Scale
  - Many production functions have increasing returns to scale for small amounts of output, constant returns for moderate amounts of output, and decreasing returns for large amounts of output.
- Graphical Analysis
  - Figure 5.5, *a* to *b*: When a firm is small, increasing labor and capital allows for gains from cooperation between workers and greater specialization of workers and equipment, so there are increasing returns to scale
  - Figure 5.5, *b* to *c*: As the firm grows, returns to scale are eventually exhausted. There are no more returns to specialization, so the production process has constant returns to scale.
  - Figure 5.5, *c* to *d*: If the firm continues to grow, the owner starts having difficulty managing everyone, so the firm suffers from decreasing returns to scale.
- **Figure 5.5 Varying Scale Economies**



5.5 Productivity and Technology Change

- Relative Productivity
  - Firms are not necessarily equally productive
  - A firm may be more productive than others if: a manager knows a better way to organize production; it's the only firm with access to a new invention; union-mandated work rules, government regulations, or other institutional restrictions affect only competitors.
  - Firms are equally productive in competitive markets, not in oligopoly markets
- Innovation
  - An advance in knowledge that allows more output to be produced with the same level of inputs is called technological progress.
  - Technological progress is neutral if more output is produced using the same ratio of inputs. It is nonneutral if it is capital saving or labor saving.
  - Organizational changes may also alter the production function and increase the amount of output produced by a given amount of inputs. In the early 1900s, Henry Ford revolutionized mass production of automobiles through interchangeable parts and the assembly line.

Managerial Solution

- Managerial Problem
  - Labor productivity during recessions
  - How much will the output produced per worker rise or fall with each additional layoff?
- Solution

– Layoffs have the positive effect of freeing up machines to be used by remaining workers. However, if layoffs force the remaining workers to perform a wide variety of tasks, the firm will lose the benefits from specialization.

– Holding capital constant, a change in the number of workers affects a firm's average product of labor. Labor productivity could rise or fall.

– For some production functions layoffs always raise labor productivity because the AP_L curve is everywhere downward sloping, for instance the Cobb-Douglass production function.

Table 5.1 Total Product, Marginal Product, and Average Product of Labor with Fixed Capital

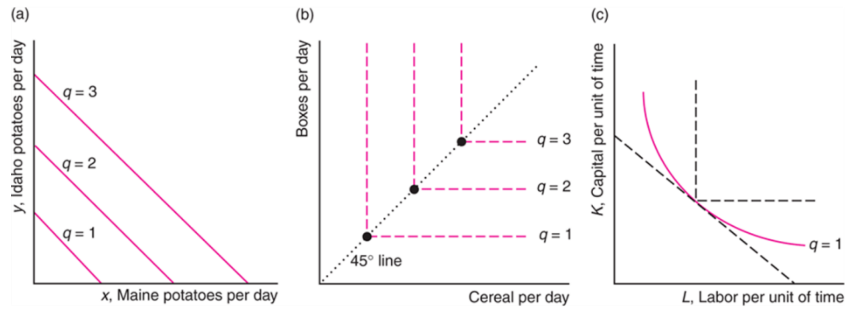
Capital, $\bar{K}$	Labor, L	Output, Total Product of Labor q	Marginal Product of Labor, $MP_L = \Delta q/\Delta L$	Average Product of Labor, $AP_L = q/L$
8	0	0		
8	1	5	5	5
8	2	18	13	9
8	3	36	18	12
8	4	56	20	14
8	5	75	19	15
8	6	90	15	15
8	7	98	8	14
8	8	104	6	13
8	9	108	4	12
8	10	110	2	11
8	11	110	0	10
8	12	108	−2	9
8	13	104	−4	8

Labor is measured in workers per day. Capital is fixed at eight fully equipped workbenches.

Table 5.2 Output Produced with Two Variable Inputs

	Labor, L					
Capital, K	**1**	**2**	**3**	**4**	**5**	**6**
1	10	14	17	20	22	**24**
2	14	20	**24**	28	32	35
3	17	**24**	30	35	39	42
4	20	28	35	40	45	49
5	22	32	39	45	50	55
6	**24**	35	42	49	55	60

Figure 5.3 Substitutability of Inputs


(a)
y, Idaho potatoes per day
q = 3
q = 2
q = 1
x, Maine potatoes per day
(b)
Boxes per day
q = 3
q = 2
q = 1
45° line
Cereal per day
(c)
K, Capital per unit of time
q = 1
L, Labor per unit of time